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EVOLUTION OF COLD DENSE NUCLEAR MATTER

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A simple analytical model is presented to describe the time evolution of cold compressed nuclear systems. The inverse meanfield method has been used. A one-dimensional three-level system is analysed in detail. Inverse methods are shown to give us the opportunity to predict the (nonlinear) evolution of cold dense nuclear systems.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Эволюция холодного сжатого ядерного вещества

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Представлена простая аналитическая модель для описания эволюции сжатых холодных ядерных систем. Использован метод обратной задачи для среднего поля ядра. Детально проанализирована одномерная трехуровневая система. Показано, что методика обратной задачи позволяет предсказать нелинейную эволюцию холодных сжатых ядерных систем.

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1. INTRODUCTION

In recent studies [1] the time evolution of initially compressed nuclear matter has been studied within the time-dependent Hartree-Fock approach. Obviously this problem is of a more general interest [2-6], so that we would like to reconsider it from a different point of view. We should also like to draw attention to the way in which hydrodynamics does predict the production of composites (or clusters), in contrast to the statement of [1], this is possible via a mechanism known for about 150 years.

Starting from hydrodynamical considerations, several groups derived, in differential ways and independently, the soliton-supporting non-

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linear Schrödinger and Korteweg — de Vries (KdV) equations at appropriate evolution equations for density distributions in nuclear matter [7-10].

A more comprehensive and general interpretation of the KdV equation emerged in connection with the application of inverse methods to the nuclear bound state problem, see [11] and references therein. This so-called inverse mean-field method (Imefim) has been demonstrated to yield useful information of nuclear radii [12], the optical potential [13], and on nuclear dynamics [14]. It has been proposed to describe also the equilibration of finite nuclear systems [15].

In this communication, based on the mean field picture, we present a simple analytical model for the time evolution of compressed nuclear matter.

2. BASIC EQUATIONS OF THE INVERSE MEAN FIELD METHOD

By now it is well known that the appropriate procedure for discussing the characteristics of nonrelativistic quantum mechanical systems is to solve the respective time-dependent many-body Schrödinger equation. However, we are not able to handle this problem adequately but for computer experiments. A prominent way of circumventing most of the associated difficulties is not resort to the physically motivated mean-field picture. It assumes that all particles of the system generate a common mean field U(x,t) in which they move rather independently. Thus, the many-body Schrödinger equation is reduced to a set of single-particle Schrödinger equations:

$$ih\frac{\partial\Psi_n}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2\Psi_n}{\partial x^2} + U\Psi_n , \qquad (1)$$

where all symbols have their usual meanings. For the sake of simplicity, below we consider only one-dimensional systems (which, are under the appropriate conditions equivalent to three-dimensional spherically symmetrical ones). Application of inverse methods and of techniques from nonlinear physics leads within Imefim to the notion to use for (conservative) time-dependent problems instead of (1) the following system of coupled equations [11]:

$$-\frac{h^2}{2m}\frac{\partial^2\Psi_n}{\partial x^2} + U\Psi_n = E_n\Psi_n , \qquad (2)$$

$$2mc \sum_{n=1}^{N} \frac{\partial}{\partial(\mathfrak{L}_n t)} U = 6U \frac{\partial U}{\partial x} - \frac{h^2}{2m} \frac{\partial^3 U}{\partial x^3} . \qquad (3)$$

The $\mathfrak{L}_n$ are constants which are determined by the initial conditions and N(n = 1,2,...,N) is the number of the bound states.

Details of the Imefim and the derivations of the resulting formulas (2,3) may be taken from /11-15/ and references.

Applying inverse methods, we have to evaluate the function K(x,y) by solving the (Gelfand — Levitan — Marchenko) integral equation /16-19/ :

$$K(x, y) + B(x + y) + \int_{x}^{\infty} B(y + z) K(x, z) dz = 0. \qquad (4)$$

The kernal B is determined by the reflection coefficients R(k) (E = $= h^2k^2/2m$) and by the N bound-state eigenvalues ($E_n = -h^2\kappa_n^2/2m$):

$$B(z) = \sum_{n=1}^{N} C_n^2(\kappa_n) \exp\left(\frac{h^2 \kappa_n^3 t}{m^2 c} - \kappa_n z\right) + \qquad (5)$$

$$+ \frac{1}{2\pi} \int_{-\infty}^{\infty} R(k) \exp\left(i \frac{h^2 k^3 t}{m^2 c} - ikz\right) dk .$$

The coefficients C_n are uniquely specified by the boundary conditions:

$$C_n(\kappa_n) = \lim_{x \to \infty} e^{\kappa_n x} \Psi_n(x, 0) \qquad (6)$$

and the wanted potential U(x,t) is given by:

$$U(x, t) = - \frac{h^2}{m} \frac{\partial}{\partial x} K(x, x) . \qquad (7)$$

By knowing all bound-state energy eigenvalues and reflection coefficients (phase-shifts), the integral equation (4) can be solved only numerically. However, in the case of reflectionless (R(k) = 0) potentials the problem is well known to have the analytical solution (for symmterical (U(x,0) = $= U(-x,0)$) potentials):

$$U(x,t) = -\frac{h^2}{m}\frac{\partial^2}{\partial x^2}\ln[\det||M||] = -\frac{2h^2}{m}\sum_{n=1}^{N}\kappa_n\Psi_n^2(x,t),$$

$$\Psi_n(x,t) = \sum_{k=1}^{N}(M^{-1})_{nk}\lambda_k(x,t),$$

$$M_{nk}(x,t) = \delta_{nk} + \frac{\lambda_k\cdot\lambda_n}{\kappa_k+\kappa_n}, \qquad (8)$$

$$\lambda_n(x,t) = C_n(\kappa_n)\exp(-\kappa_n x + \frac{h^2\kappa_n^3 t}{m^2 c}),$$

$$C_n(\kappa_n) = \sqrt{2\kappa_n\prod_{k\neq n}^{N}\left|\frac{\kappa_n+\kappa_k}{\kappa_n-\kappa_k}\right|}.$$

A glance at (8) shows that wave functions, potential and densities are uniquely determined by N energy eigenvalues.

3. THREE-LEVEL SYSTEM

Although there difinitely is some progress in the application of inverse methods to nuclear physics [11-15,21], they are not yet too popular. As an illustration for the work of these methods, we consider, in this section, a one-dimensional three-level system in detail. Simplest one-level and two-level systems [20] have been analysed earlier. A three-level system may be useful for modelling the evolution of light nuclei, for instance, of oxygen [1].

Let us present the main formulas to calculate wave functions of the three-level system $(\kappa_3 > \kappa_2 > \kappa_1)$:

$$\Psi_1(x,t) = (2\kappa_1(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1})(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}))^{1/2} D^{-1}(x,t)\times$$

$$\times(\mathrm{ch}(\xi_2+\xi_3) - (\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2})\,\mathrm{ch}(\xi_3-\xi_2)), \qquad (9)$$

$$\Psi_2(x,t) = (2\kappa_2 (\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1})(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}))^{1/2} D^{-1}(x,t)\,(\mathrm{sh}(\xi_3+\xi_1) +$$

$$+ (\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1})\,\mathrm{sh}(\xi_3-\xi_1)),$$

$$\Psi_3(x,t) = (2\kappa_3 (\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2})(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}))^{1/2} D^{-1}(x,t) \times$$

$$\times (\mathrm{ch}(\xi_2+\xi_1) + (\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1})\,\mathrm{ch}(\xi_2-\xi_1)),$$

$$D(x,t) = \mathrm{ch}(\xi_1+\xi_2+\xi_3) + (\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1})(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1})\mathrm{ch}(\xi_3+\xi_2-\xi_1) +$$

$$+ (\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1})(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2})\,\mathrm{ch}(\xi_3+\xi_1-\xi_2) +$$

$$+ (\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1})(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2})\,\mathrm{ch}(\xi_2+\xi_1-\xi_3),$$

$$\xi_n(x,t) \equiv \kappa_b x - \frac{h^2\kappa_n^3 t}{m^2 c}, \quad n = 1,2,3. \tag{9}$$

The asymptotic behaviour of the wave functions and the potential has the following form (ξ_n is fixed):

$$\lim_{t\to\infty} \Psi_k(x,t) = \begin{cases} \sqrt{\kappa_n}\,\mathrm{sech}(\xi_n - \xi_n^\circ), & k = n \\ 0, & k \neq n \end{cases}$$

$$\lim_{t\to\infty} U(x,t) = -\frac{h^2\kappa_n^2}{m}\,\mathrm{sech}^2(\xi_n - \xi_n^\circ), \tag{10}$$

$$\xi_n^\circ = \frac{1}{2}\ln\Big(\prod_{k=1}^{n-1}(\frac{\kappa_n-\kappa_k}{\kappa_n+\kappa_k})^2 \prod_{m\neq n}^{N} |\frac{\kappa_n+\kappa_m}{\kappa_n-\kappa_m}|\Big).$$

So, for large x and t the time-dependent one-body potential and density distribution are represented by a set of solitary waves. The energy spectrum of an initially compressed system completely determines widths, velocities and phase shifts (ξ_n°) of the solitons.

In (8-10) the constants $\mathfrak{L}_n$ entering into (3) should be calculated from the ground state spectrum for a given nuclear system. It leads to a renormalization of the velocities of the solitons. The qualitative picture of the evolution remains unchanged.

The initially compressed system expands so that for large times one observes separate density solitons. This picture is in accordance with the time-dependent Hartree-Fock simulation of the time evolution of a compressed ^{16}O nucleus /1/. The disassembly shows collective flow and clusterization.

4. SUMMARY

Attention has been drawn to the fact that inverse methods lead to a simple analytical model for the qualitative time evolution of cold compressed nuclear metter. The exact numerical solution could even be applied for a quantitative treatment.

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